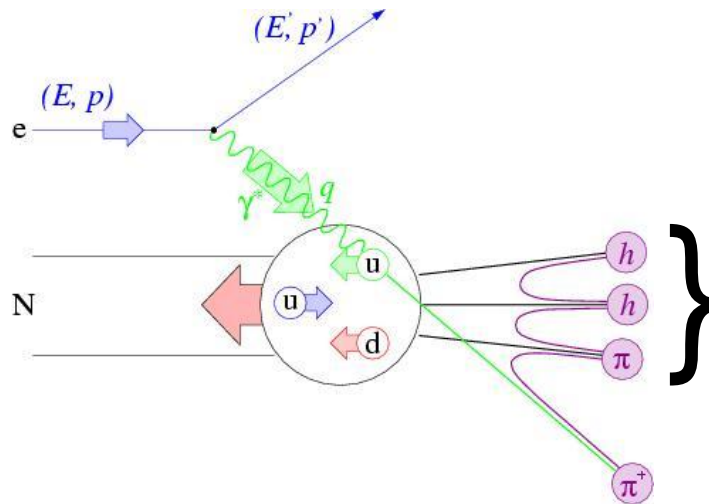


# The Quark-Parton Model and Low-Energy Factorization Studies in Semi-Inclusive Pion Electroproduction

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For Jlab Hall-C E00-108 Collaboration

# Flavor Decomposition through semi-inclusive DIS



**(e,e')**  $M_x^2 = W^2 = M^2 + Q^2 (1/x - 1)$

DIS probes only the sum of quarks and anti-quarks  $\rightarrow$  requires assumptions on the role of sea quarks  $\sum e_q^2 (q + \bar{q})$

**Solution: Detect a final state hadron in addition to scattered electron**

**(e,e'm)**  $M_x'^2 = W'^2 = M^2 + Q^2 (1/x - 1)(1 - z)$

**SIDIS  $\rightarrow$  Can 'tag' the flavor of the struck quark by measuring the hadrons produced: 'flavor tagging'**

$$\frac{1}{\sigma} \frac{d\sigma}{dz} (ep \rightarrow hX) = \frac{\sum_q e_q^2 f_q(x) D_q^h(z)}{\sum_q e_q^2 f_q(x)}$$

$f_q(x)$ : parton distribution function

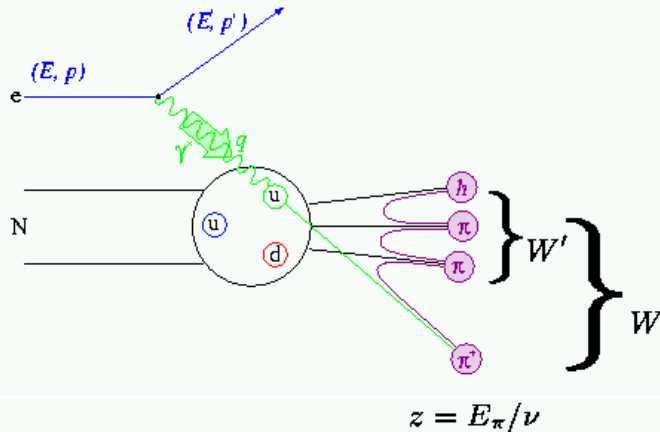
$D_q^h(z)$ : fragmentation function

Measure inclusive (e,e') at same time as (e,e'h)

$z = E_m/\nu$

# Meson Electroproduction in Semi-Inclusive DIS

$$e + N \rightarrow e' + \pi + X$$



$$z = \frac{E_h}{\nu} \quad x = \frac{Q^2}{2m\nu} \quad \nu = E - E'$$

$$W^2 = m_p^2 + Q^2 \cdot \left( \frac{1}{x} - 1 \right)$$

$$W'^2 = m_p^2 + Q^2 \cdot (1 - z) \cdot \left( \frac{1}{x} - 1 \right)$$

- **At high energies the SIDIS process factorizes into a hard virtual photon-quark interaction and a subsequent quark hadronization**
- **The cross section can be decomposed as a product of quark distribution functions and fragmentation function:  $\sigma \sim f(x, Q^2) \cdot D(z)$**
- **A consequence of this factorization is independence of the hard scattering process of  $z$  and hadronization of quark momentum  $x$**
- **At low energies a more efficient description interaction of hadrons in terms of collective degrees of freedom, the mesons and hadrons**
- **At low energies, this factorization ansatz is expected to break down, due to effects of FSI, resonant excitations and high twist.**

# Experiment E00-108 in JLab Hall C

**The Goal:** To verify, for the first time, the existence of quark-hadron duality and low-energy factorization

$$e + p \longrightarrow e' + \pi^\pm + X$$

$$e + D \longrightarrow e' + \pi^\pm + X$$

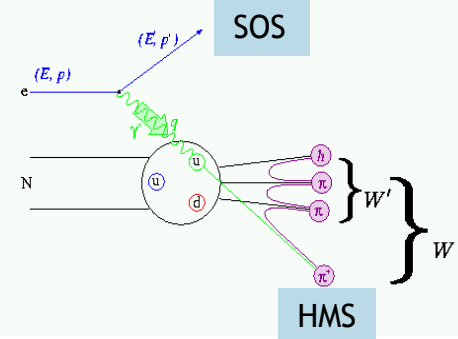
- Coincidence: HMS + SOS**



- HMS to detect pions ( $\pi^-$  or  $\pi^+$ )**
- SOS to detect electrons**
- 4 cm LH2 and LD2 targets**
- Al (dummy) to estimate cryo target wall contribution**
- Beam energy 5.5 GeV**
- Beam current 20-70  $\mu\text{A}$**

- The experiment run in 2003**

- z-dependence ( $z=0.3-1.0$ ) at  $x=0.3$**
- x-dependence ( $x=0.25-0.6$ ) at  $z=0.55$**
- $\theta_\pi$  ( $0^\circ-8^\circ$ ) at fixed  $z=0.55$  and  $x=0.3$ .**



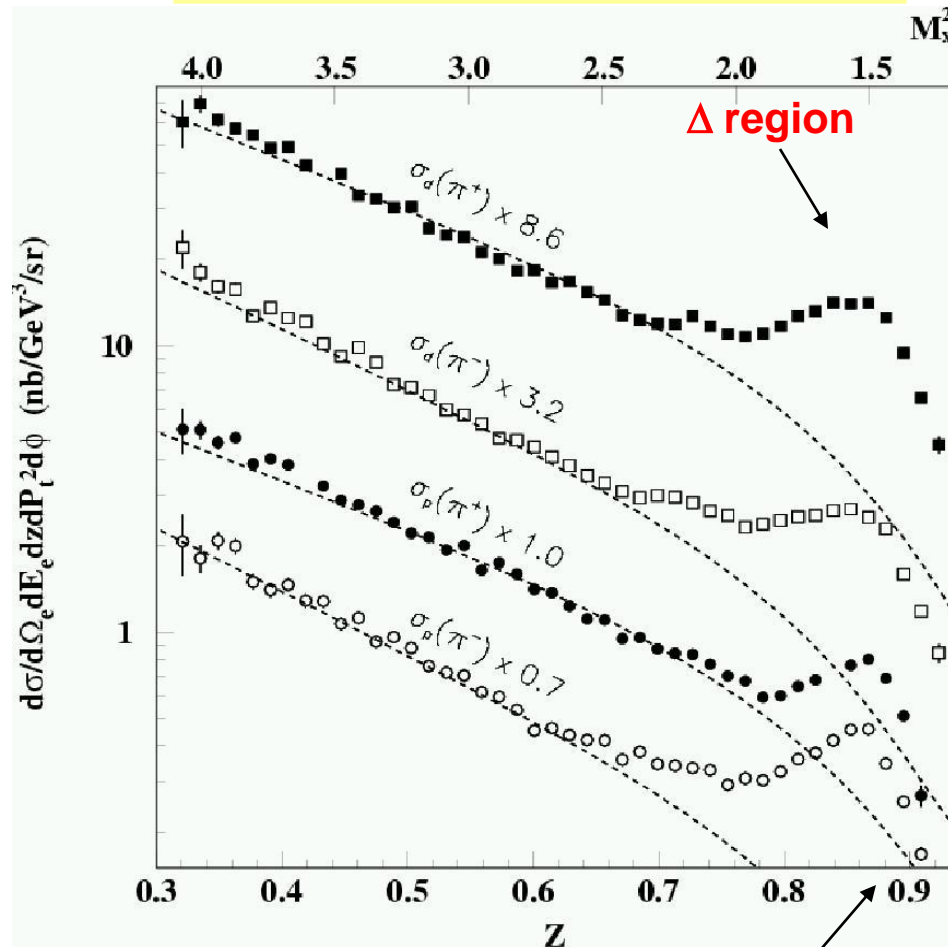
$$x=Q^2/2M\nu$$

$$z=E_\pi/\nu$$

**Semi-Inclusive  $\pi^\pm$  electroproduction to the region  $M_x \sim 1.5 \text{ GeV}$**

# How Can We Verify Factorization?

## Z-Dependence of cross section



$(m_{\Delta}^2 \approx 1.5 \text{ GeV}^2)$

$$\sigma \sim \sum e_q^2 q(x) D_q^{\pi}(z)$$

factorization

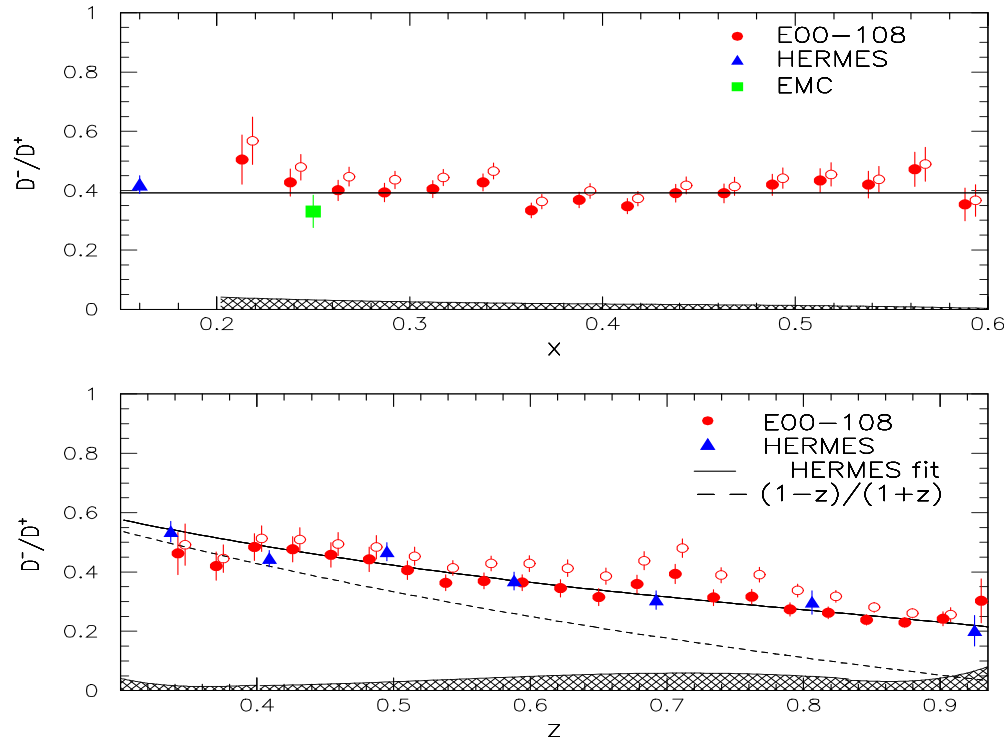
- Good agreement between our data and simple quark-parton model for  $z < 0.65$  (assuming factorization)
- Large excess in the data at  $Z > 0.8$  with respect to the prediction reflects the  $N-\Delta$
- Within our kinematics ( $P_t \sim 0, x=0.3, Q^2=2.30 \text{ GeV}^2$ ),  $M_x^2$  directly related to  $z$ :

$$W'^2 = m_p^2 + Q^2 \cdot \left( \frac{1}{x} - 1 \right) (1 - z)$$

$$\rightarrow W'^2 \equiv M_x^2 \sim (1 - z)$$

# How Can We Verify Factorization?

## X & Z-Dependence of $D^-/D^+$



## $D^-/D^+$ from Deuteron $\pi^+$ to $\pi^-$ ratio

$$\sigma_d^{\pi^+} \propto (4D^+ + D^-)(u + d)$$

$$\sigma_d^{\pi^-} \propto (4D^- + D^+)(u + d)$$

$$\frac{\sigma_d^{\pi^+}}{\sigma_d^{\pi^-}} = \frac{4D^+ + D^-}{4D^- + D^+}$$

$$D^-/D^+ = (4 - r) / (4r - 1)$$

$$r = \sigma_d(\pi^+) / \sigma_d(\pi^-)$$

$$z = E_\pi / \nu$$

Closed (open) symbols reflect data after (before) events from coherent p subtracted.

The hatched area indicates the systematic uncertainty.

- Near-independence from  $x$ , as expected
- Our results agree with HERMES & EMC
- The resonant contribution at  $z > 0.8$  cancel out ! (Close & Isgur)
- Our data far exceed the Feynman and Field expectation (dashed line)

# How Can We Verify Factorization?

Neglect sea quarks and assume no  $p_t$  dependence to parton distribution functions

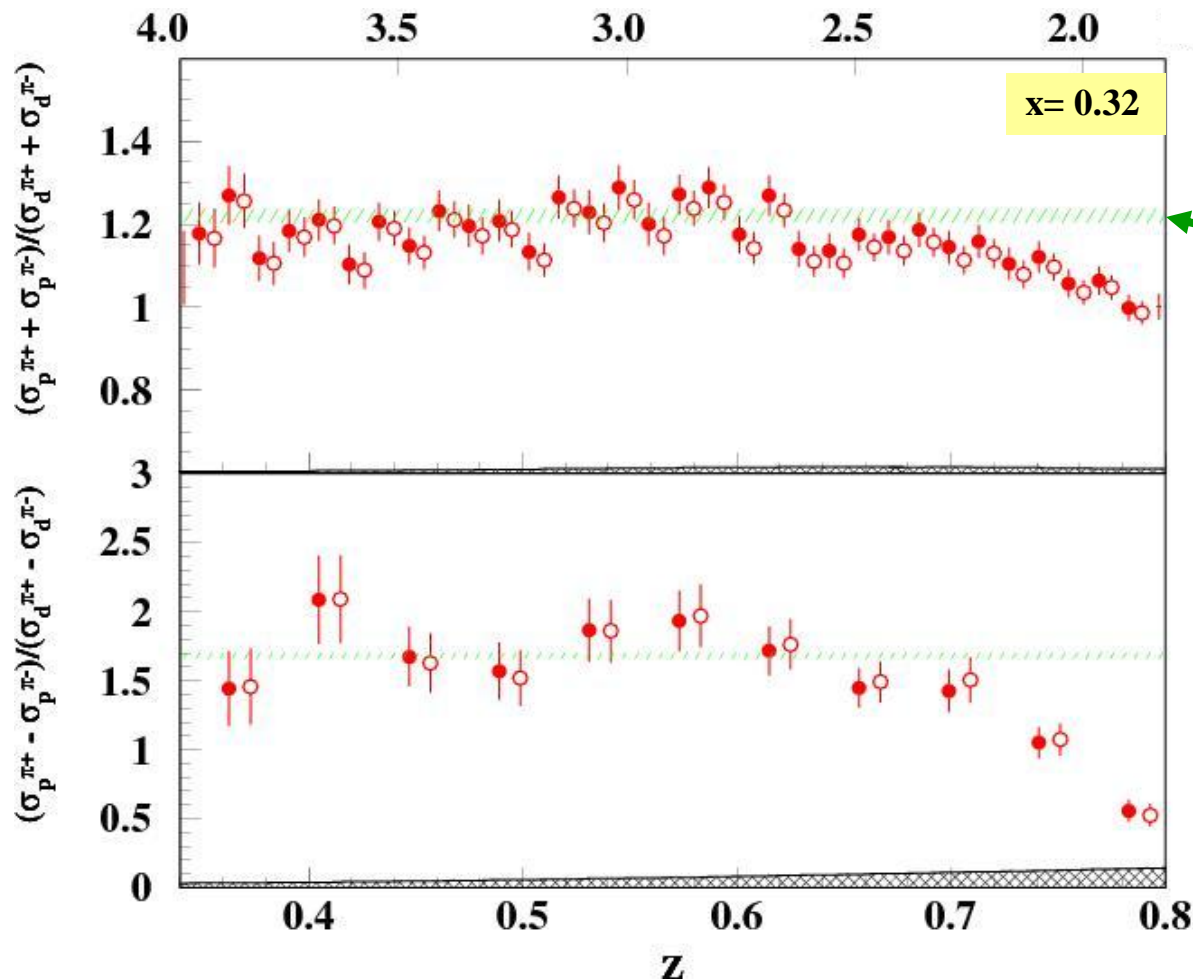
→ Fragmentation function dependence drops out in Leading Order

$$\begin{aligned} & \sigma_p(\pi^+) + \sigma_p(\pi^-) / [\sigma_d(\pi^+) + \sigma_d(\pi^-)] \\ & \sim [4u(x) + d(x)] / [5(u(x) + d(x))] \\ & \sim \sigma_p / \sigma_d \quad \text{independent of } z \text{ and } p_t \end{aligned}$$

$$\begin{aligned} & \sigma_p(\pi^+) - \sigma_p(\pi^-) / [\sigma_d(\pi^+) - \sigma_d(\pi^-)] \\ & = [4u_v(x) - d_v(x)] / [3(u_v(x) + d_v(x))] \\ & \quad \text{independent of } z \text{ and } p_t, \\ & \quad \text{but more sensitive to assumptions} \end{aligned}$$



# How Can We Verify Factorization?



$$\sum_q e_q^2 q(x) D_{q \rightarrow M}(z)$$

**GRV & CTEQ,  
@ LO or NLO**

Leading-Order  
x-z factorization

**Good description for  
Proton and deuteron  
targets for  
 $0.4 < z < 0.65$**

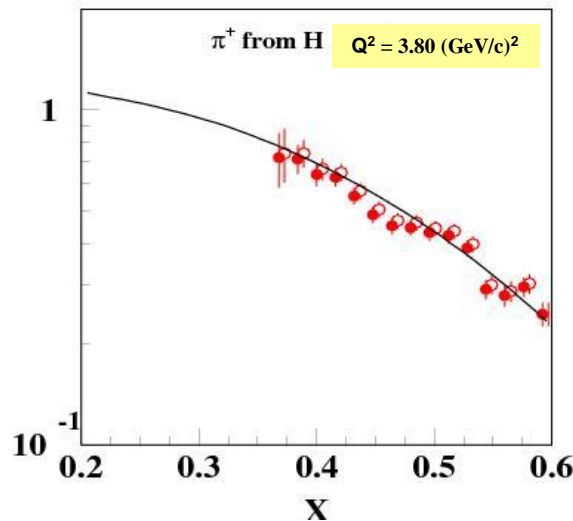
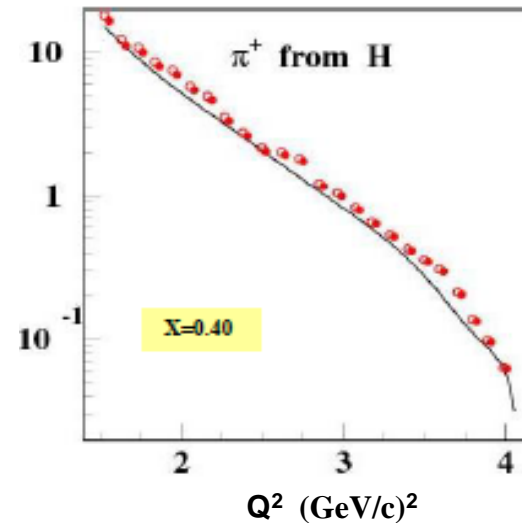
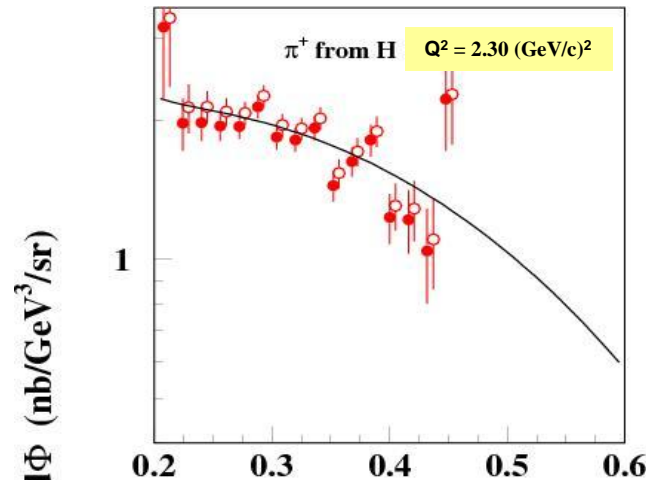
*(Note: for  $z = 0.65$   
 $M_x^2 \approx 2.5 \text{ GeV}^2$ )*

For  $z < 0.7$  ratios are independent of  $z$  as expected from quark-model

Closed (open) symbols are data after (before) events from  $\rho$  production are subtracted



# x and Q<sup>2</sup> dependence of the cross sections



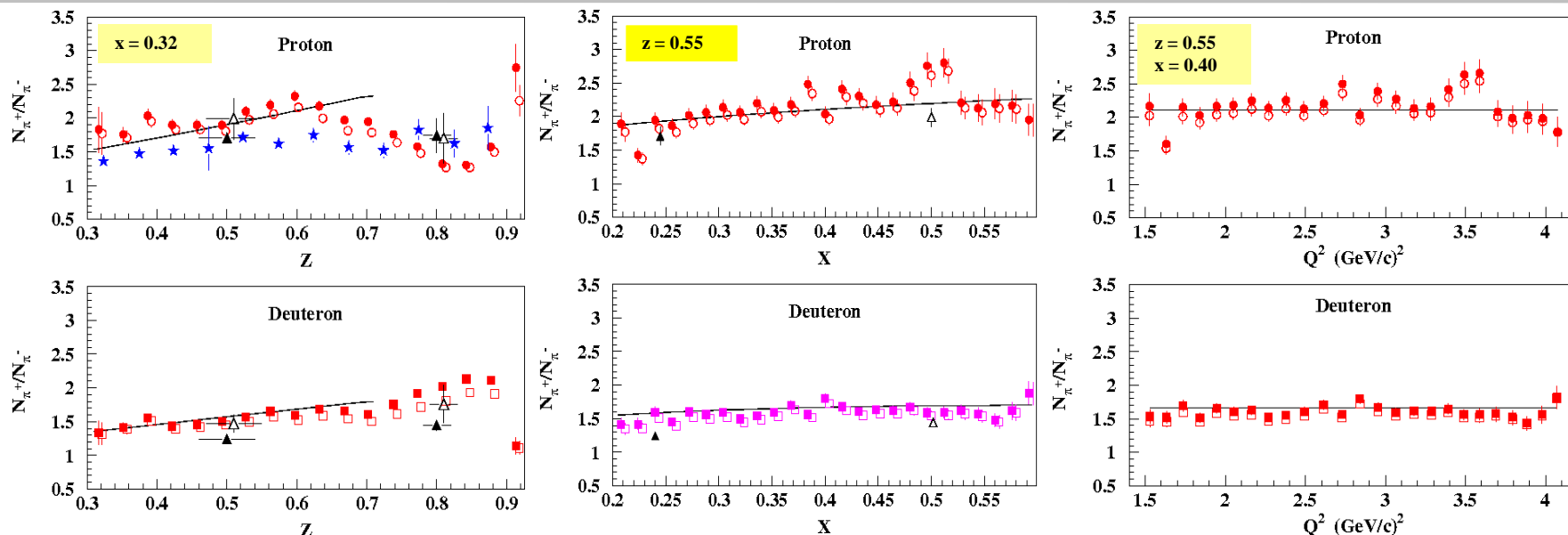
Solid curve is a naive parton model calculation assuming CTEQ5M parton distribution functions at NLO and BKK fragmentation functions

Overall, one finds good agreement, but there are detailed differences in the cross section measurements (also for the deuterium target)

Solid (open) symbols are data after (before) subtraction of events from diffractive  $\rho$

E00-108: Onset of the Parton Model

# z, x and Q<sup>2</sup> dependence of the $\pi^+/\pi^-$ ratios



**z-dependence:**  $\pi^+/\pi^-$  ratios from proton is larger than those reported by HERMES (stars), but agrees with the Cornell data (triangles). At  $0.65 < z < 0.85$ , the ratio decreases because  $\pi^-\Delta^{++}$  cross section is larger than  $\pi^+\Delta^0$ . The sharp rise at  $z > 0.85$  is due to exclusive  $\pi^+$  production. The  $\pi^+/\pi^-$  ratio from deuteron agrees with quark-parton model expectation

**x-dependence:**  $\pi^+/\pi^-$  ratios from proton and deuteron targets agrees with the Cornell data (triangles), they also in good agreement with the quark-parton model prediction.

**Q<sup>2</sup>-dependence:** The  $\pi^+/\pi^-$  ratios in agreement with the quark-parton model predictions.

# The Ratio of d/u Valence Quarks

The valence quark ratio can be extracted from a combination of the proton and deuteron  $\pi^\pm$  cross section as:

$$\sigma_p^{\pi^+} \sim D^+(4u + \bar{d}) + D^-(4\bar{u} + d)$$

$$\sigma_p^{\pi^-} \sim D^-(4u + \bar{d}) + D^+(4\bar{u} + d)$$

$$\sigma_D^{\pi^+} \sim (4D^+ + D^-)(u + d) + (4D^- + D^+)(\bar{u} + \bar{d})$$

$$\sigma_D^{\pi^-} \sim (4D^- + D^+)(u + d) + (4D^+ + D^-)(\bar{u} + \bar{d})$$

$$\sigma_p^{\pi^+} - \sigma_p^{\pi^-} \sim (D^+ - D^-)(4u_v - d_v) \quad u_v = u - \bar{u}$$

$$\sigma_D^{\pi^+} - \sigma_D^{\pi^-} \sim (D^+ - D^-)(3u_v + 3d_v) \quad d_v = d - \bar{d}$$

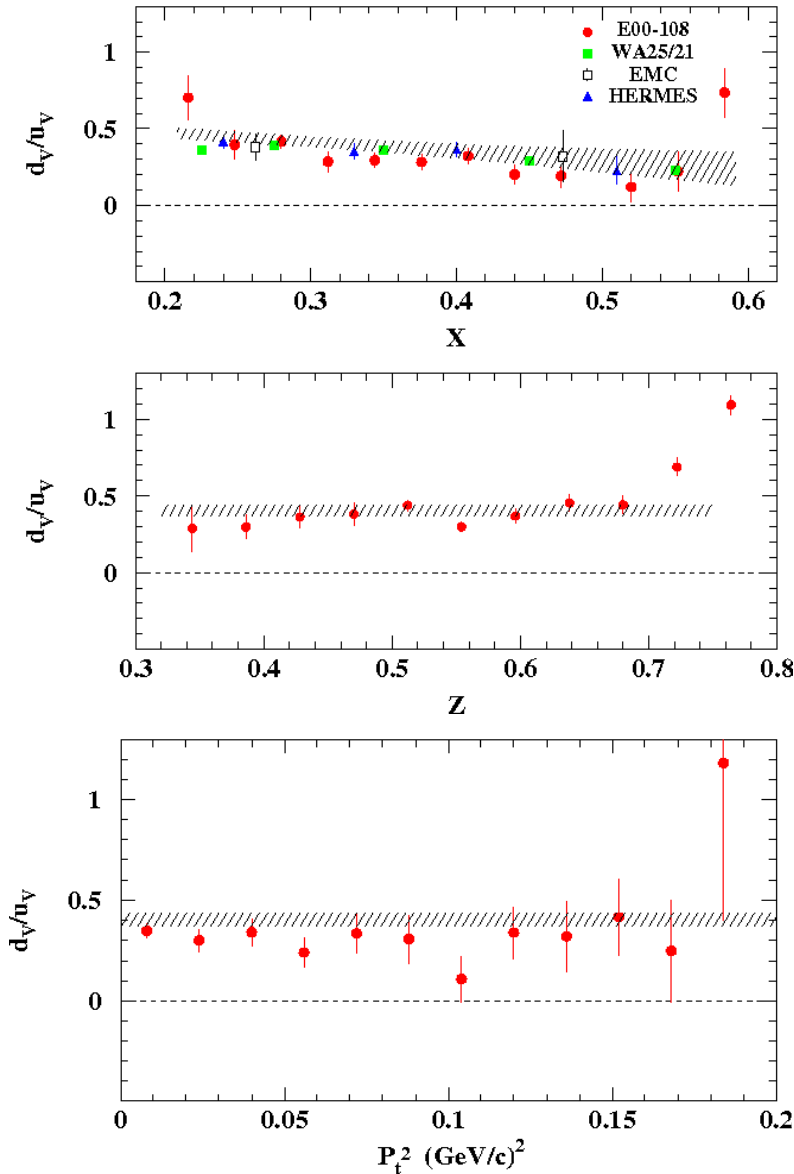
$$R_{pD}^- = \frac{\sigma_p^{\pi^+} - \sigma_p^{\pi^-}}{\sigma_D^{\pi^+} - \sigma_D^{\pi^-}} = \frac{4u_v - d_v}{3(u_v + d_v)} \quad d_v/u_v = \frac{4 - 3R_{pD}^-}{3(R_{pD}^- + 1)}$$

**x-dependence of our data (low energy and  $P_t \sim 0$ ) are in good agreement with WA21/25 and EMC results, and slightly low as compared to the CTEQ expectations.**

**z-dependence of the ratio will be indication for violation of the isospin symmetry and factorization assumption**

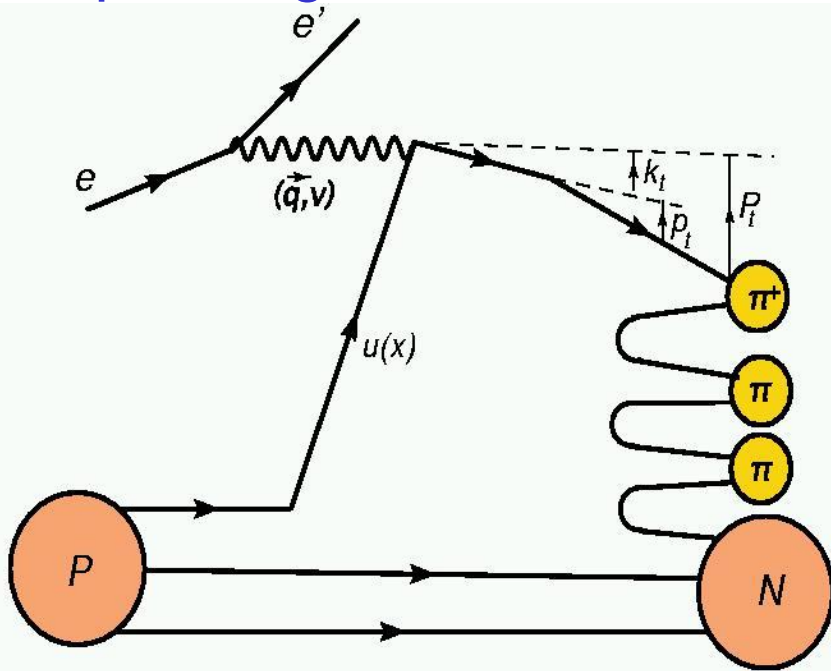
**A sharp increase of the ratio at  $z > 0.7$  corresponds in our kinematics to missing mass  $< 2.5 \text{ GeV}^2$ , where the resonances contribution become dominant.**

**Within our statistical accuracy we can not rule out any  $P_t$  dependence of the valence quark ratio.**



# Transverse Momentum Dependence of SIDIS

- Not much is known about the orbital motion of partons
- Significant net orbital angular momentum of valence quarks implies significant transverse momentum of quarks



Final transverse momentum of the detected pion  $P_t$  arises from convolution of the struck quark transverse momentum  $k_t$  with the transverse momentum generated during the fragmentation  $p_t$ .

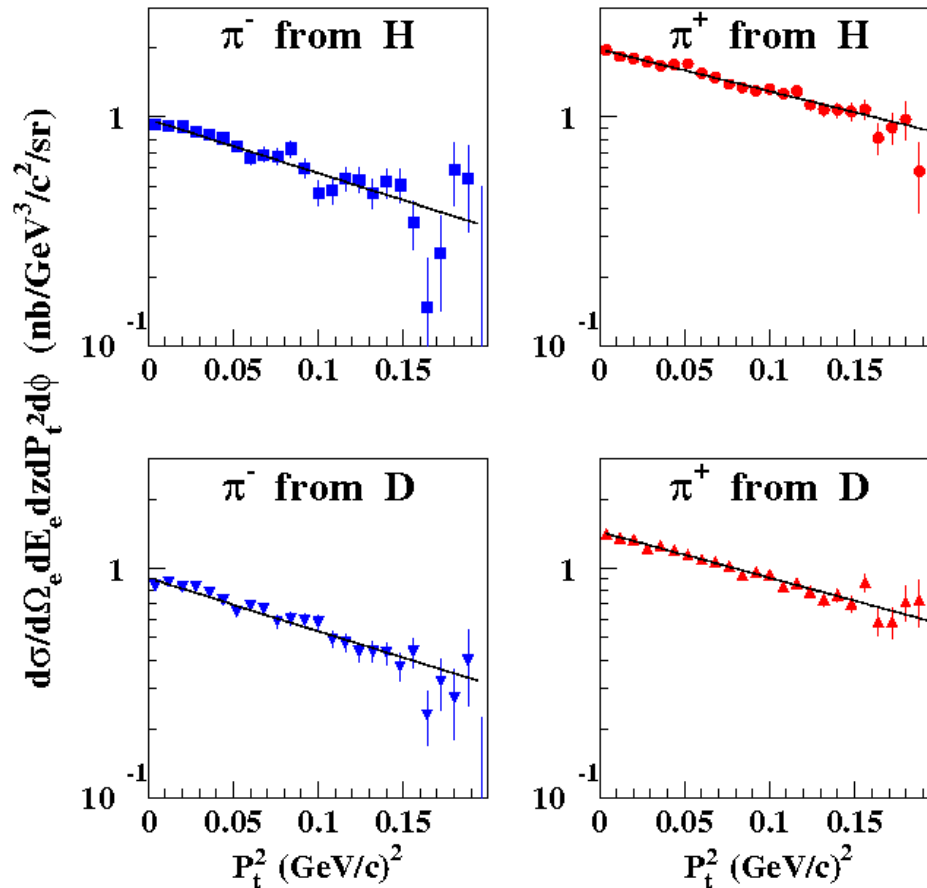
$$P_t \approx p_t + z k_t + O(k_t^2/Q^2)$$

$$z = E_\pi/v$$

$p_t \sim \Lambda < 0.5 \text{ GeV}$  optimal for studies as theoretical framework for Semi-Inclusive IDIS has been well developed at small transverse momentum

[A. Bacchetta et al., JHEP 0702 (2007) 093].

# $P_t$ dependence of SIDIS Cross Sections



$$\sigma_{\text{SIDIS}} \sim \sigma_{\text{DIS}} (dN/dz) b \exp(-bP_t^2)$$

$$b_{H^-} = 5.44 \pm 0.36$$

$$b_{D^-} = 5.35 \pm 0.26$$

$$b_{H^+} = 4.24 \pm 0.17$$

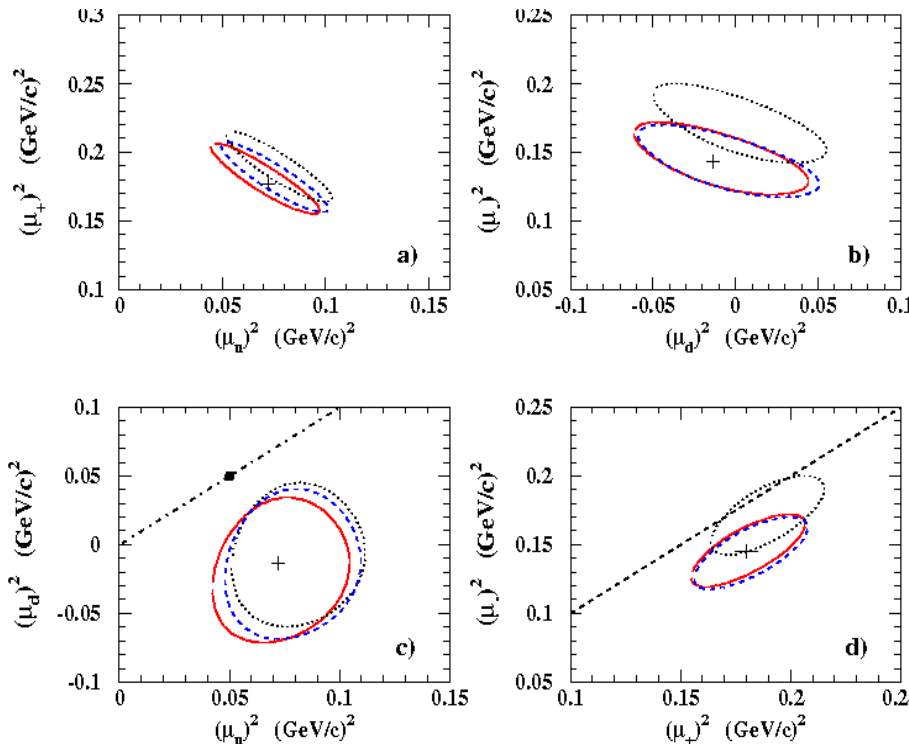
$$b_{D^+} = 4.64 \pm 0.17$$

- $P_t^2$  dependence for  $\pi^-$  or  $\pi^+$  are similar for H & D targets, but not identical
- Slope for the  $\pi^+$  are smaller than for the  $\pi^-$  (for H and D)
- We study the data in the context of quark parton model, assuming:  $P_t = p_t + zk_t$

- We assume the width quark and fragmentation functions are Gaussian in  $k_t$  and  $p_t$ , and  $\langle P_t^2 \rangle = \langle k_t^2 \rangle + \langle p_t^2 \rangle$ . We introduce separate width for u and d quarks, and separate width for favored ( $D^+$ ) and unfavored ( $D^-$ ) fragmentation functions
- We assume:  $b_u^\pm = (z^2 \mu_u^2 + \mu_\pm^2)^{-1}$  and  $b_d^\pm = (z^2 \mu_d^2 + \mu_\pm^2)^{-1}$

# Quarks & Fragmentation Functions Transverse Momentum

$$\sigma_{\text{SIDIS}} \sim \sigma_{\text{DIS}} (dN/dz) b \exp(-bP_t^2) \Rightarrow b \rightarrow b_q^\pm (b_u^\pm \text{ \& } b_d^\pm), \text{ and } b_q^\pm = (z^2\mu_q^2 + \mu_\pm^2)^{-1}$$



$$\sigma_p^{\pi^+} = C[4c_1 \cdot \exp(-b_u^+ P_t^2) + (d/u)(D^-/D^+)c_2 \cdot \exp(-b_d^- P_t^2)]$$

$$\sigma_p^{\pi^-} = C[4(D^-/D^+)c_3 \cdot \exp(-b_u^- P_t^2) + (d/u)c_4 \cdot \exp(-b_d^+ P_t^2)]$$

$$\sigma_n^{\pi^+} = C[4(d/u)c_4 \cdot \exp(-b_d^+ P_t^2) + (D^-/D^+)c_3 \cdot \exp(-b_u^- P_t^2)]$$

$$\sigma_n^{\pi^-} = C[4(d/u)(D^-/D^+)c_2 \cdot \exp(-b_d^- P_t^2) + c_1 \cdot \exp(-b_u^+ P_t^2)]$$

**Fit values:**

- $D^-/D^+ = 0.43 \pm 0.01$ ;  $d/u = 0.39 \pm 0.03$
- $\mu_u^2 = 0.07 \pm 0.03 \text{ GeV}^2$   $\mu_d^2 = 0.0 \pm 0.05 \text{ GeV}^2$
- $\mu_+^2 = 0.18 \pm 0.02 \text{ GeV}^2$   $\mu_-^2 = 0.14 \pm 0.02 \text{ GeV}^2$

- **Fit results for  $D^-/D^+$  agree with HERMES (0.42), and  $d/u$  ratio with LO GRV (0.40)**
- **Fit tends to larger  $k_t$  width for  $u$  quarks than for  $d$  ( $\mu_d^2 \sim 0$ )**
- **Fragmentation width  $\mu_+$  and  $\mu_-$  are similar (as predicted by Anselmino)**

# Summary (1)

- We have measured semi-inclusive  $\pi^\pm$  electroproduction on proton and deuteron targets, using 5.5 GeV energy  $e^-$  beam at JLab.
- For the first time the low-energy factorization of cross section, and phenomenon of quark-hadron duality have been observed at low  $W^2$
- Several ratios constructed from these data exhibit, provided that  $W^2 > 4.0 \text{ GeV}^2$  and  $z < 0.7$ , the features of factorization in a sequential electron-quark scattering, and a quark-pion fragmentation process.
- We find the azimuthal dependence of the data to be small, as compared to the typically larger azimuthal dependences found in exclusive pion electroproduction data, but consistent with data from other groups and theoretical expectations based on SIDIS.
- In a context of a simple model with only valence quarks and only two fragmentation functions, we find the  $k_t$  width of u quark to be larger than for d quarks, for which the width is consistent with zero.



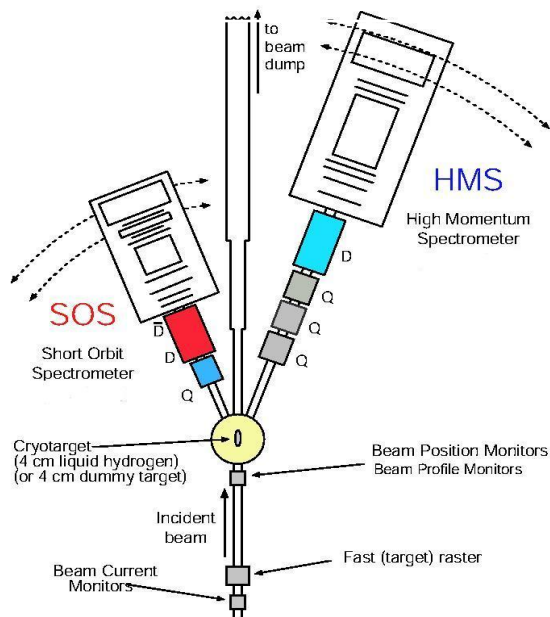
# Summary (2)

- We find that the favored and unfavored fragmentation  $p_t$  width are similar to each other, and both larger than the two quark widths.
- We have shown the sensitivity of our results to be small to possible corrections due to both radiative events from exclusive pion production channels and pions originating from diffractive  $\rho$  decay.
- In many cases, the corrections are negligible, although they can become large at large values of  $z$ .
- E00-108 results can only be considered suggestive at best: (limited kinematic coverage and very simple model assumptions).
- Our results call for a more sophisticated theoretical analysis based on future precision cross section and ratio measurements over a large range of kinematics ( $Q^2$ ,  $P_t$  and  $\cos\phi$ ).
- We believe our results will provide a fruitful basis for future studies of the quark-parton model and more sophisticated model calculations at relatively low energies.

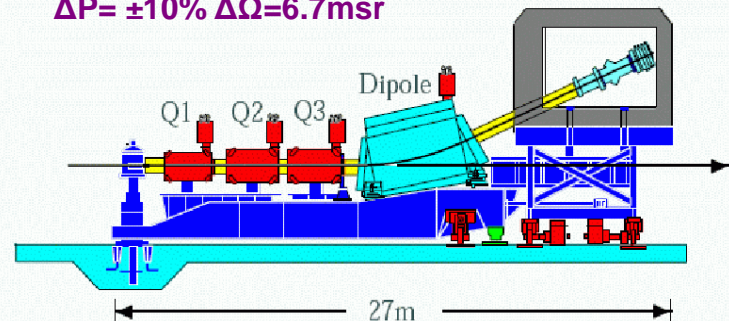
# BACKUP SLIDES

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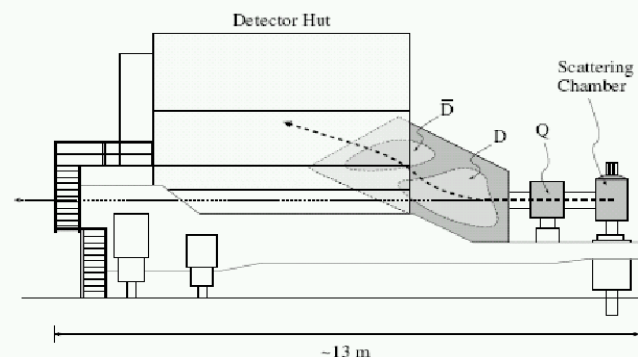
# Hall C Basic Experimental Setup



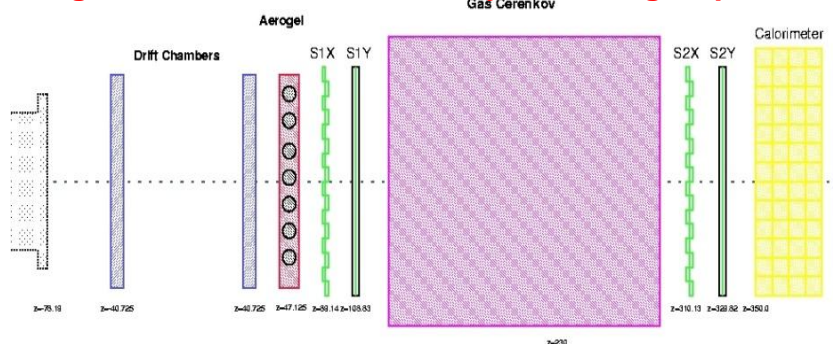
**High Momentum Spectrometer:  $P_{\text{max}} = 7.4 \text{ GeV/c}$**   
 $\Delta P = \pm 10\%$   $\Delta \Omega = 6.7 \text{ msr}$



**Short Orbit Spectrometer:  $P_{\text{max}} = 1.74 \text{ GeV/c}$**   
 $\Delta P = \pm 20\%$   $\Delta \Omega = 7.5 \text{ msr}$



**Aerogel and Calorimeter Built by Yerevan group**

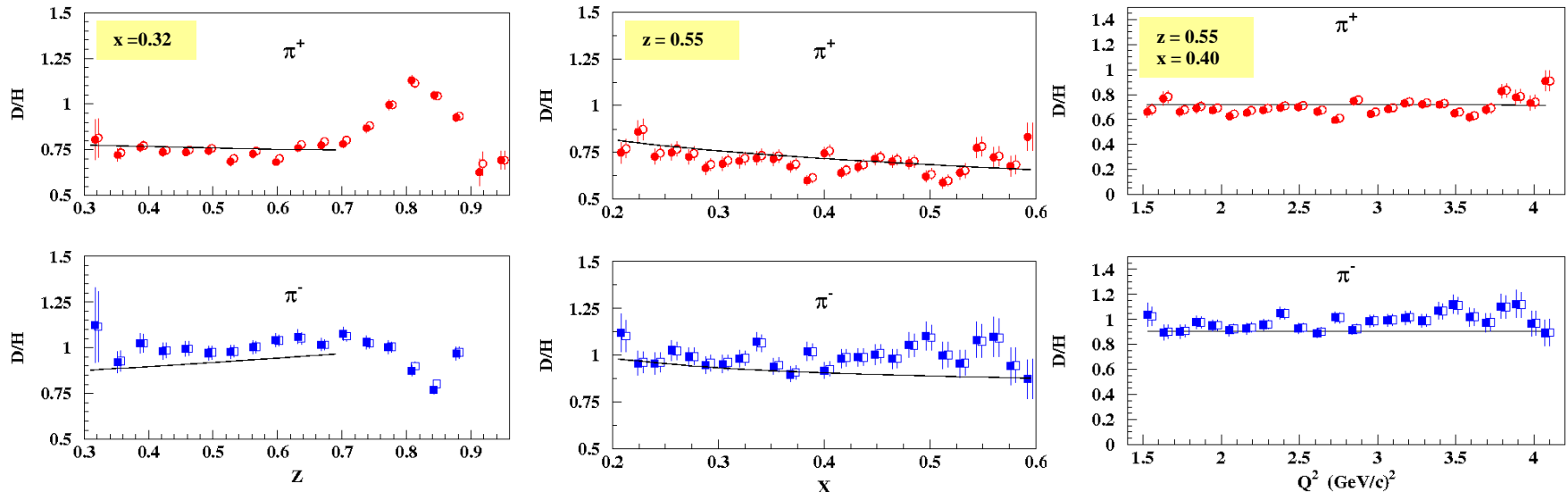


-- Momentum resolution:  $< 0.1\%$

-- Angular resolution:  $\sim 2.0 \text{ mrad}$

← Very similar detector packages

# z, x and Q<sup>2</sup> dependence of the D/H ratios

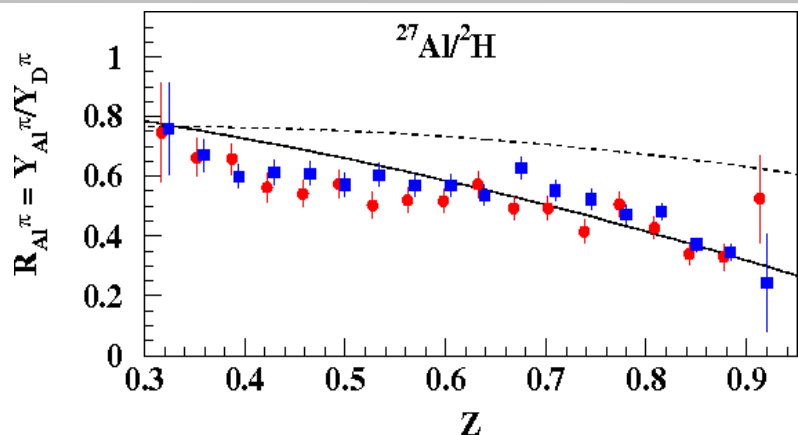


**z-dependence:** The ratios show relatively flat behavior as a function of  $z$  and reasonable good agreement with the quark-parton model calculations up to  $z \approx 0.7$ . In the region  $z > 0.7$  we again noticed effects from the N- $\Delta$  transition.

**x-dependence:** The x-dependence of the ratios is remarkably close to the quark-parton model prediction, but absolute magnitudes slightly differs from the model estimates.

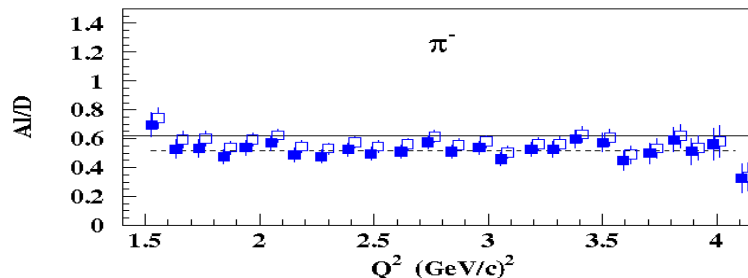
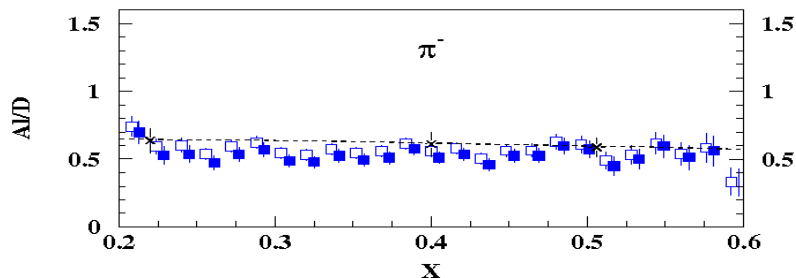
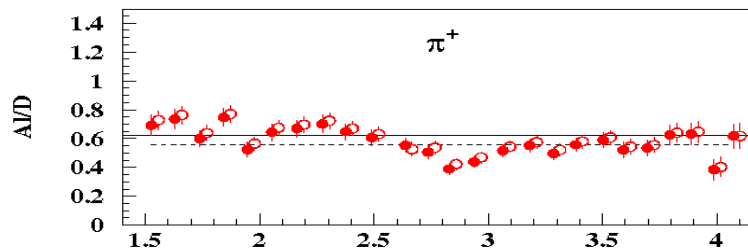
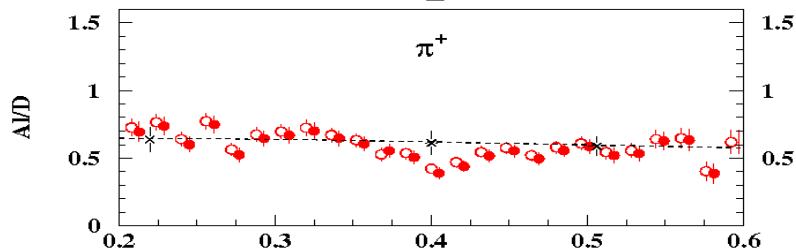
**Q<sup>2</sup>-dependence:** The D/H ratios in good agreement with the quark-parton model expectations.

# z, x and Q<sup>2</sup> dependence of the A/D ratios



The A/D ratio for both  $\pi^+$  (red circles) and  $\pi^-$  (blue squares) are below unity and decreasing with  $z$ , similar to other's data

The ratios agree with gluon radiation theory (solid curve), and low than string tension model expectation (dashed cure).

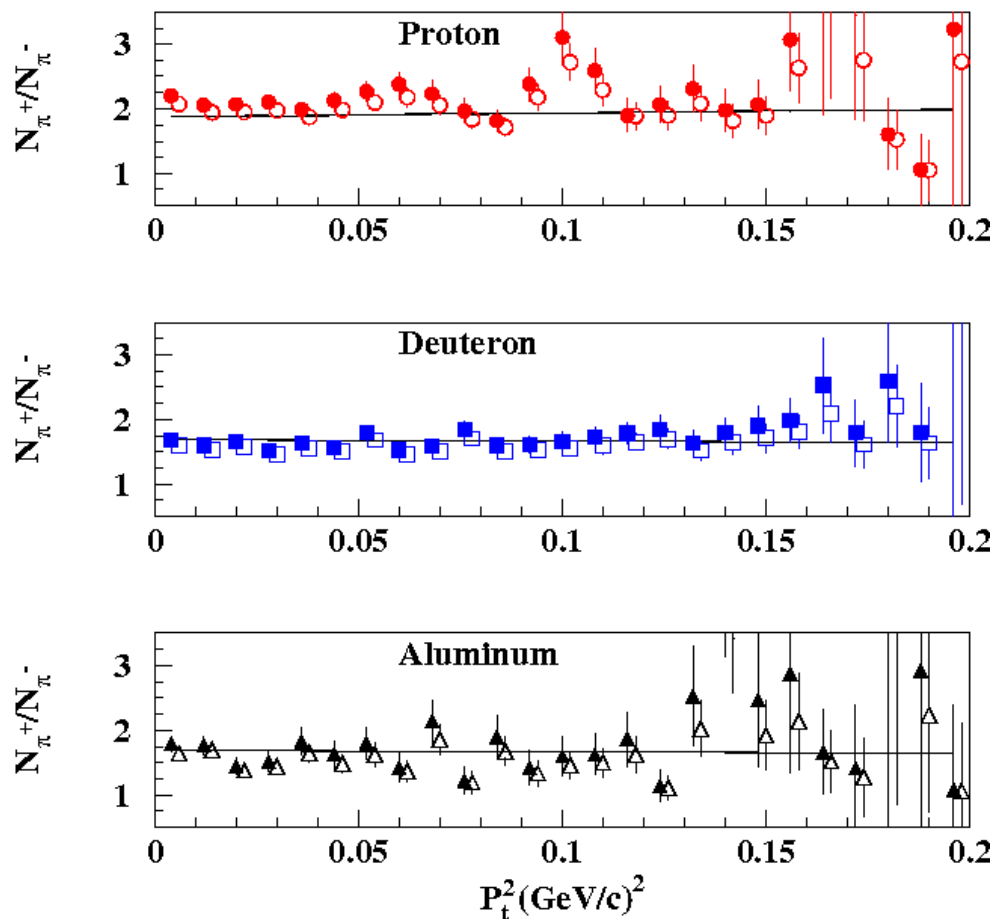


The  $x$ -dependence of the A/D ratios are weak.

The ratios agree with SLAC data (cross symbols), and with SLAC A-dependent EMS fit (dashed lines).

The  $Q^2$ -dependence of the ratios are flat with average value  $\sim 0.56$  (dashed lines), which is low than expected 0.62 from the gluon radiation theory (solid line).

# $P_t^2$ -dependence of $\pi^+/\pi^-$ ratios for H, D and Al

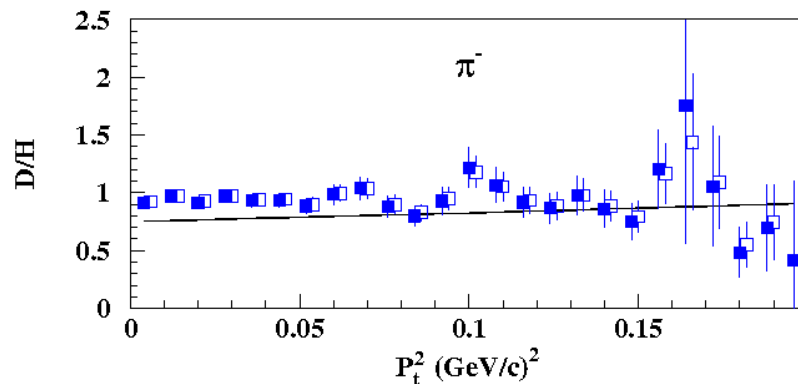
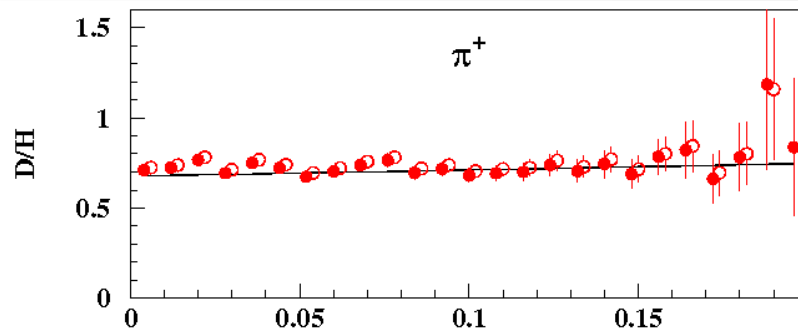


$z = 0.55$   
 $x = 0.32$

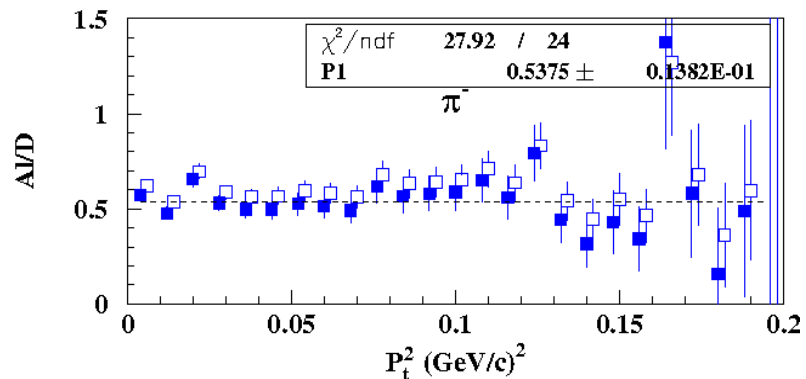
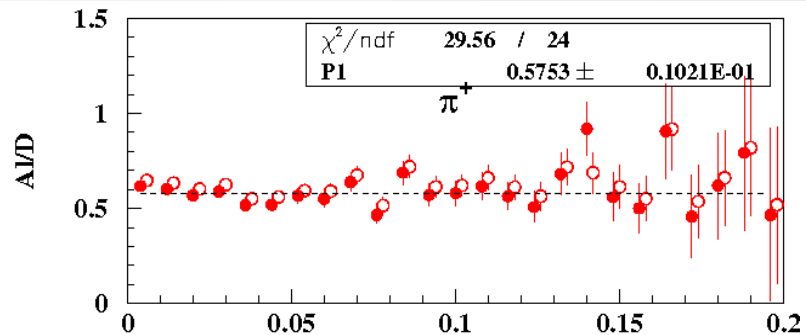
- The average values of the  $\pi^+/\pi^-$  ratios for D and Al targets are smaller than that for the protons
- The  $P_t^2$  dependence for all three targets nearly are flat

- The Solid lines are simple quark-parton model expectations
- Solid (open) symbols are data after (before) events from  $\rho$  are subtracted

# $P_t^2$ -dependence of D/H and A/D $\pi^+/\pi^-$ ratios



- The D/H ratio for  $\pi^+$  are in good agreement with the quark-parton model prediction (solid line).
- For  $\pi^-$  the experimental data on average  $\sim 10$ - $15\%$  higher relative to the model expectation.



- The A/D ratio nearly constant with  $P_t^2$  (dashed lines) and are slightly different for  $\pi^+$  and  $\pi^-$ .
- The data show that the A/D ratio is reduced at high  $z$ , as was observed by HERMES group.